Stress Lines in Beams of Unsymmetrical Cross Section

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STRESS LINES IN BEAMS OF UNSYMMETRICAL CROSS SECTION.

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Following up the method already made use of in treating of Stress and Strain in beams of symmetrical cross section,* it is proposed in the present article to extend the same methods to beams of unsymmetrical cross section. As a typical beam of this character, the Channel with web vertical will be first considered.

As a preliminary to the treatment of the Channel Beam, a brief résumé of a few of the facts brought out in the articles mentioned may be useful. These facts may be summarized as follows: —

1. The horizontal unit shear at any point of the beam is always accompanied by an equal right-angled vertical (or transverse) unit shear.

2. The total horizontal shear at any point is equal to the difference between the horizontal forces acting on the two

* "Stress and Strain." Proc. Del. Co. Inst. Sci. Vol. VIII, No. 1, May, 1916.

"Stress and Deformation in the I Beam." Proc. Del. Co. Inst. Sci. Vol. VIII, No. 2, December, 1916.

ends of the elementary "block" of material lying above or beyond the element of length under consideration.

3. Passing from the middle towards the ends along any level line, the above horizontal forces are all "spilled off" in the shape of horizontal shear, becoming reduced to zero at the ends. The horizontal forces become less and less by regular decrements or subtractions from a previous value. The horizontal shear usually increases towards the ends, but not by additions to a previous value.

4. The horizontal shear may be regarded as the physical lever-arm of the couples in the upper and lower part of the beam. Without this shear the parts of the beam would not work together as a harmonious "resisting moment."

5. In the I Beam the compression in the right-hand edge of the upper flange, for instance, must work in conjunction with its most remote comrade, the tension in the left-hand edge of the lower flange. This can only be through the medium of some kind of shear. Hence we must assume the existence of horizontal shear in the flange as well as in the web. The horizontal shear in the flanges, however, has its shearing planes vertically disposed, while in the web the shearing planes are horizontal, Figure 1.

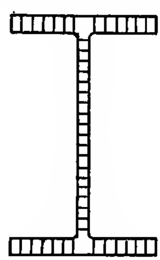

FIGURE 1

As a definite problem for the application of these and other principles to beams of unsymmetrical cross section, let

us take up the determination of the general character of the stresses and deformations in the Channel Beam, set with web vertical and loaded uniformly. For the present let us assume this loading to be in a narrow line along the web, so as to avoid consideration of the bending of the flange by the direct load.

In order to balance the external forces and reactions, each element of the body will be acted upon by certain forces which for convenience we will resolve vertically, transversely and longitudinally. These are the resultants of the various external forces acting upon the body, and necessary to equilibrium. By resolving the forces acting upon the elements in various directions the actual direction and amount of the true internal stresses acting in the material may be found. In the present article we will consider more particularly the direction and characteristics of the lines of maximum stress rather than the absolute value of the stresses.

As the Channel stands on edge subjected to its load, we know it is acted upon by vertical shear and bending moment. Further, we know that if a vertical unit shearing stress exists at any point, an equal and opposite longitudinal unit shearing stress must exist also, as required by the principles of statics. This opposing longitudinal unit shear is produced by or results from the decrements in the value of the horizontal direct stresses already mentioned. Hence, each element of the body must be acted upon by two equal right-angled shearing unit stress. As a consequence of the bending moment we know also that each element must in addition be acted upon by direct stress, tension or compression.

As far as the web is concerned these are the same forces as are met with in a beam of rectangular cross section.

In the flanges we meet a rather different array of forces, similar to those already mentioned in connection with the I Beam. A longitudinal shear, with vertically arranged shearing planes must exist in order to transfer the decrements of longitudiual stresses to the web and thence by ordinary

horizontal shear to the neutral axis. Along with this longitudinal shear in the flange there must also exist its right-angled companion shear likewise with vertical shearing planes.

As regards the variation in value of the stresses in a given cross section of the Channel, it is evident that in the web the direct stress varies as the distance from neutral surface and the horizontal and vertical shear vary inversely as the square of the distance from the neutral surface. It is also evident that in the flanges the direct stresses are independent of the distance from web and are constant, while the longitudinal and cross shears vary inversely as the distance from the web.

Knowing the resultant forces acting upon the elements of the body, it is easy to predict the characteristics of the actual lines of maximum internal stress. In Figure 2 are indicated the forces above enumerated which the elements must resist, and the nature of the resulting lines of maximum stress. The formulas for tracing these lines may be found in any advanced work on the mechanics of materials, and are

$$\text{Direct Stress: } \cot 2\,\theta = -\frac{s}{2\,v} \quad \ldots\ldots\ldots\ (1)$$

$$\text{Shear: } \tan 2\,\phi = \frac{s}{2\,v} \quad \ldots\ldots\ldots\ (2)$$

Where θ = angle of direct stress with longitudinal direction
ϕ = angle of shearing plane with longitudinal direction
s = direct unit stress, tension positive, compression negative
v = unit longitudinal shear

The figure shows respectively top view, elevation and bottom view of the beam. The various forces are represented by arrows or by the letters T for tension, C for compression and S for shear.

So far the stress lines as determined are based upon statical principles. When the Channel deflects under its loads these lines of stress will vary to a small extent. Under

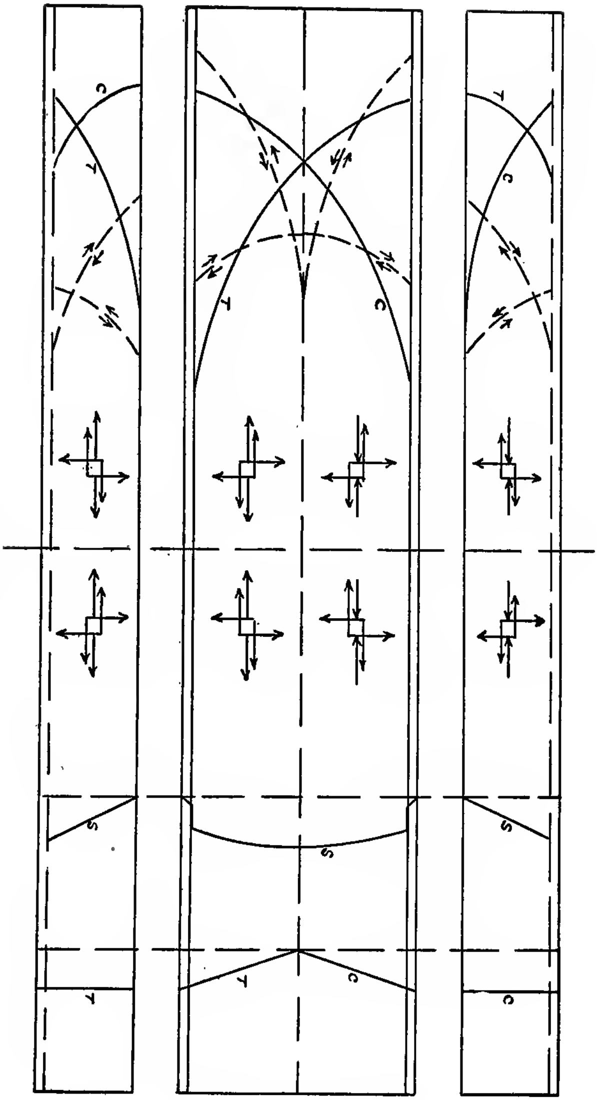

FIGURE 2

deflection the upper portion of the beam expands in all directions, while the lower portion contracts. This means that a transverse unit tension in all directions must exist in the upper portion of the beam and a transverse unit compression in the lower portion. These forces may be resolved vertically and horizontally and their effect considered. The vertical components in the web and the horizontal forces in the flanges are those which will act to modify the lines of maximum stress as already found. The actual effect, of course, is small, and if the deformation were known these forces might be added to those of Figure 2 before making the resolution.

Another effect of the expansion and contraction just mentioned is to raise the centre of gravity of the section and consequently the neutral axis. This alters the amount of the horizontal forces, producing another modification in the lines of maximum stresses. These modifications in the stress lines are naturally small, and for all practical purposes may be neglected.

Let us now consider the Channel used as a beam with web horizontal and flanges vertical, for example, turned upward. The position of the neutral plane will usually be in the lower part of the flanges. The lines of maximum stress in the flanges will be calculated in the usual way, and will have the general characteristics shown in Figure 3. The figure also suggests the forces acting on the elements of the body and the manner in which they vary in value from top to bottom.

The lines of direct stress cut the upper edges of the flanges at 90° and 0°, and they cross the neutral axis at 45°. They do not, however, intercept the upper surface of the web at 90° and 0°, since the horizontal shear does not fall to zero. The lines as sketched are only meant to indicate the general characteristics of the curves.

The lines of shear in the flanges cut the upper edge of flange at 45°, and the neutral axis at 90° and 0°. They do not cut the upper surface of the web at 45°, because

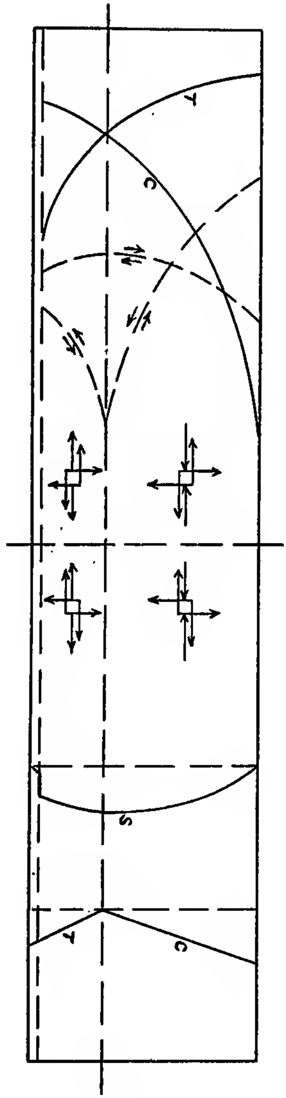

FIGURE 3

the horizontal shear, as before, does not reduce to zero.

As regards the lines of stress in the web, another method of reasoning must be used. In the first place, we will encounter longitudinal shear with vertical shearing planes, as only in this way can the two flanges be tied together to work in harmony with the web. The planes of shearing for the whole section are more or less as indicated in Figure 4.

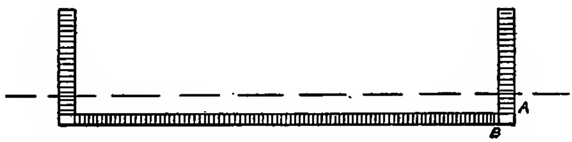

FIGURE 4

The distribution of forces acting on the elements of the body in the four quadrants of the web and their variation in value are indicated in Figure 5.

The resolution of these forces will give lines of maximum internal stress for the plane of the web after the manner shown to the left in the figure.

The lines of direct stress cut the middle line of the beam at 0° and 90°. They cut the edges of the web at angles differing from 45° a certain amount, since the ratio $-\frac{s}{2v}$ does not reduce to zero.

The lines of shear cut the middle line at 45° and approach 0° and 90° at the edges of the web.

As mentioned, the lines of stress cut the boundaries in certain cases at angles differing more or less from limiting values. These angles may be found by substitution of the proper values in Formulas (1) and (2). They will depend, of course, upon the relative values of the unit direct stress and unit horizontal shear.

If the flange and web are of same thickness, the shear and direct stress, at A and B, Figure 4, have nearly the same

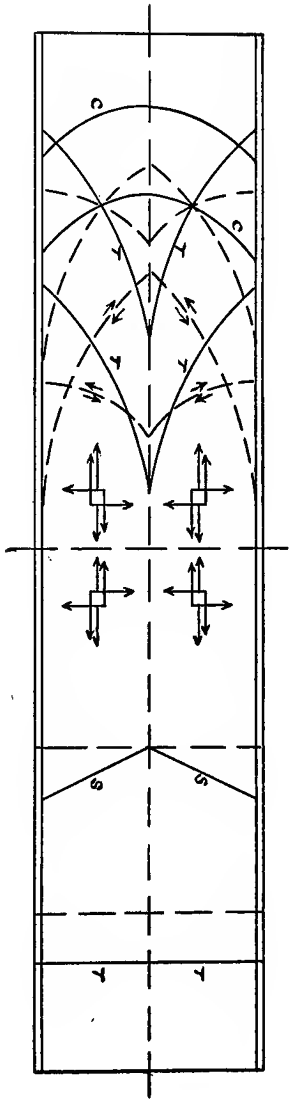

FIGURE 5

values. The maximum stress lines in the web would, therefore, cut the junction of web and flange at nearly the same angles as the corresponding lines in the flange. As commonly the web is thinner than the flange, it results that the unit longitudinal shear on plane B is greater than on plane A, which means a different angle of cutting. A consideration of Formulas (1) and (2) will show whether the lines in the web will make a greater or less angle with the longitudinal direction than the corresponding lines in the flanges.

In case the Channel is used the other side up, that is, flanges directed downwards, the same line of reasoning may be used and lines corresponding to those already shown may be easily sketched. We will leave it to the fertile imagination of the reader to picture them.

Another common unsymmetrical cross section to examine is that of the Angle Beam. We will consider it in one position only, for example, horizontal leg on top, and vertical leg directed downwards, referring to same as flange and web respectively. Since the Angle is to all intents nothing more than part of a Channel, the nature of its stresses may be easily indicated by reference to the previous figures. In the Angle Beam the shearing planes will be similar to those in the web and upper half flange of the I Beam or in the web and upper flange of Channel Beam on edge. The maximum longitudinal shear will occur at the neutral axis, which of course passes through the centre of gravity of the section.

The lines of maximum stress in the web of the Angle will resemble those of the flanges of the Channel Beam laid flat. Figure 3 inverted and with stresses appropriately reversed will indicate them very well.

In the flange, looked at from above, the lines can be easily shown to have the characteristics of the corresponding lines in the upper flange of the Channel Beam on edge, Figure 2, upper sketch.

In beams of thin cross section with the parts vertically and horizontally disposed, it is not difficult, as we have seen,

to determine the characteristics of the lines of maximum internal stress. In the foregoing the underlying principles have been sufficiently developed to clearly exemplify the method of treatment. In the case of thick cross sections, other than the rectangular, the matter is by no means so simple. In a subsequent article it is expected to treat of circular, triangular and other cross sections.